Darmstadt Discussion Papers in ECONOMICS

TECHNISCHE UNIVERSITÄT DARMSTADT

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Nr. 231

Arbeitspapiere der Volkswirtschaftlichen Fachgebiete der TU Darmstadt

ISSN: 1438-2733

ISSN 1438-2733

Printed in Germany
Technische Universität Darmstadt
Department of Law and Economics
D – 64289 Darmstadt
Germany
www.wi.tu-darmstadt.de

A VAR Evaluation of Classical Growth Theory*

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March 28, 2018

Abstract

Over the past two decades, there have been numerous attempts in economic theory to model the historical regime of a Malthusian trap as well as the transition to growth in one coherent framework, or in other words, a unified growth theory. However, in most of these models, an important effect suggested by Malthus has been frequently omitted. By including what he had called "the great preventive check" in the traditional Malthusian model which is based on the principle of population, the principle of diminishing returns and the principle of labor division, the transition can be modelled in a very simple dynamic macroeconomic framework. The aim of this paper is to first construct and calibrate the suggested classical model and to eventually employ a conventional VAR–Method to provide evidence of the above principles using country–specific annual historical data on crude birth rate, crude death rate and GDP per capita growth rate. As a result, it is argued that emerging economies follow a universal macroeconomic pattern of development. A decreasing death rate is succeeded by a decreasing birth rate which at the same time induces GDP per capita to rise sustainably. The correspondingly advanced microeconomic theory suggests that increasing life expectancy tends to create a demographic structure that is much less prone to overpopulation.

JEL classification: B12, C32, J11, O11

Keywords: Demographic Transition, Malthusian Trap, Unified Growth Theory, Classical Growth Theory, Vectorautoregression

*I would like to thank Uwe Hassler for valuable methodological support as well as Volker Caspari, Guenther Rehme and Bertram Schefold for very helpful theoretical comments. Besides, I am greatly indebted to Heinrich Ursprung and Ralf Brueggemann for their decisively inspiring suggestions.

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1 Introduction

1.1 The Economic Problem

Over the past two hundred years, the world has seen unprecedented growth rates in terms of gross domestic product (GDP) per capita, albeit very unevenly distributed across countries and regions. In the year 1820, world population had amounted to approximately 1.1 billion. In the same year, the value of all goods produced was estimated by Maddison (2006) at about USD 700 billion, measured in 1990 USD. By the year 2003, population had grown to around 6.4 billion people, while GDP was calculated by the UN to lay in the range of USD 40,900 billion. Correspondingly, GDP per capita had risen tenfold from approximately USD 640 to USD 6,400. Although the reconstruction of historical data determining past living standards has been subject to some debate, it is obvious that economic growth cannot have increased over the last few thousand years at the same speed as it did over the last two hundred years. A re–projection of those growth rates would result in "absurdly low" living conditions during medieval times.[1] It is therefore plausible to presume a pre–modern era of stagnation or at least very slow growth that Keynes (1930) had characterized as follows.

> From the earliest times of which we have record–back, say, to two thousand years before Christ — down to the beginning of the eighteenth century, there was no very great change in the standard of life of the average man living in the civilised centres of the earth.[2]

By that time, he was well aware of the fact that roughly since the beginning of the English Industrial Revolution the world economy had begun to experience a transitional phase from stagnation to growth, optimistically concluding that

> assuming no important wars and no important increase in population, the economic problem may be solved, or be at least within sight of solution, within a hundred years.[3]

Among others, Clark (2009) took up on the "economic problem" and collected historical data of GDP per capita illustrating the transition from a historical regime of stagnation to a regime of growth for the case of Great Britain in the form of the well-known "hockey

[1] Mokyr and Voth (2010), p. 8.
[2] Keynes (1930), p. 1.
[3] ibid., p. 4.

stick".[4] These data do not simply reflect British economic history, but can be globally generalized in so far as every economy once found itself or currently is located in a regime of stagnation. However, roughly at the beginning of the 19th century something changed, as England had apparently become the first economy to generate sustained economic growth.[5] Reluctantly at first, then progressively catching up, the major part of the world economy followed the English example. As Broadberry & O'Rourke (2010) put it,

> viewed in the grand sweep of history, this change was undoubtedly radical, and must be ranked alongside other epoch–making changes such as the change from hunting and gathering to settled agriculture.[6]

Recently, North (2013) argued that the elucidation of the transition to growth seems to be "the most important historical question that might conceivably be possible to answer".[7] Building on these assessments, the primary object of this work is to disentangle the effects that made for an era of stagnation and those enabling the transition to growth, or to use Keynes' wording, to solve the economic problem.

Having introduced the economic problem of stagnation and growth, the rest of the work is structured as follows. First, a set of stylized facts will be offered as a touchstone for unified growth models. Secondly, a new interpretation of the classical growth model is suggested to be capable of integrating the mechanisms of stagnation and growth. More specifically, the mechanisms will rely on the operation of four general principles that have partly been incorporated into neoclassical theory, while other parts seem to have disappeared along with classical theory. In order to arrive at an empirically testable macroeconomic growth model, first, the propositions will be translated qualitatively into causal relationships. Then, these relationships will be quantitatively defined in a system of linear equations, exemplarily calibrated and simulated to show that the classical model can indeed account for the stylized facts of stagnation and growth. After having checked the validity of the classical model with regard to the stylized facts, the third chapter deals with the empirical identification of the classical short–run

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[4] See figure 4.1, app. I. The general form of the time series has been confirmed by Allen's (2001) data series on real wages of London laborers, which move, in accordance with economic theory, in the long run proportionally to GDP per capita (see figure 4.2, app. I).

[5] The date 1800 is often chosen to mark the British "take–off".

[6] Broadberry & O'Rourke (2010), p. 1.

[7] As Lucas (1988), p. 5, put it, "The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think of anything else."

mechanisms between demographic and economic variables. To this end, a vectorautoregression is estimated and impulse response functions are employed to find evidence of the causal corresponding relationships. The work concludes with the finding that the economic principles classical growth theory was built upon are found to prevail globally.

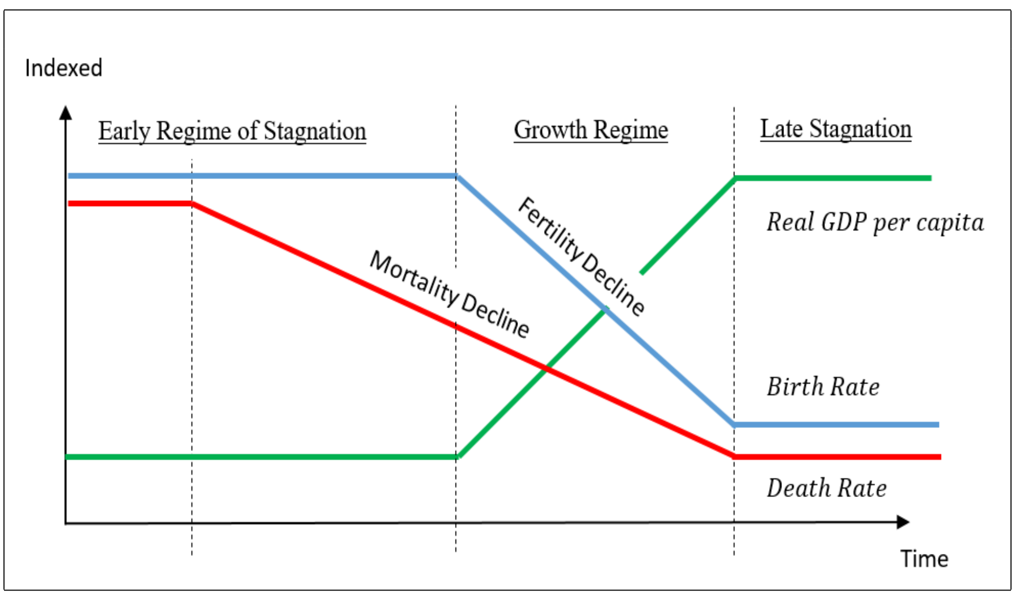
1.2 The Stylized Facts of Stagnation and Growth

Mokyr and Voth (2010) summarized the development of the theoretical literature on stagnation and growth by stating that

> from the 1990s onwards, scholars started to search for an overarching theory that could encompass both slow growth and the transition to rapidly increasing per capita incomes — a "unified growth model". The field has flourished since. [8]

The most comprehensive recent elaborations on the stylized facts of stagnation and growth are probably found in Clark (2007) and Galor (2011), who suspect a causal link between the demographic transition and the breakout from the Malthusian trap. The

Figure 1.1: Stylized facts of stagnation and growth.



focus on demographic variables is certainly not surprising given that the mechanism of stagnation is generally regarded to rely on a “population trap”. Following the stylized

[8] Mokyr and Voth (2010), p. 8.

time series illustrated in figure 1.1, the subsequent set of stylized facts regarding the demographic transition and the transition from stagnation to growth are viewed to be sustained by the data. Firstly, during the transition to growth, there is some evidence of causality running from demographic to economic variables. There is no modern economy in which GDP per capita increased sustainably that has not gone through a demographic transition. Secondly, the model of the demographic transition gives some evidence of death rates positively affecting birth rates. There is no modern economy in which a sustainable decrease in the birth rate preceded a decrease in the death rate. Thirdly, the mortality decline was not initiated by an increase in GDP per capita, but by a — from an economic point of view — rather exogenously determined epidemiological transition. Appendix I provides examples in accordance with these stylized facts pictured in figure 1.1.

2 Classical Unified Growth Theory

When constructing a unified growth model, it is usually suggested that population growth formerly seemed to outperform growth in production, causing stagnation, whereas in more recent times population growth is observed to have slowed down, offering the potential for economic growth. The aim of this chapter is to build a mathematical framework of macroeconomic short–run mechanisms that can account for these stylized facts.

2.1 Historical Background

> The Malthusian law of population is one of the great achievements of thought. Together with the principle of the division of labor it provided the foundations of modern biology and for the theory of evolution; [...] the objections raised against the Malthusian law as well as against the law of [diminishing] returns are vain and trivial. Both laws are indisputable.[9]

The interactions between population and production have attracted scientific interest for many centuries. Political considerations certainly contributed to suppressing "the population question" from entering economic theory during the 20th century. Nonetheless, North (2013) reminded his audience that the origins of the question "why did the

[9] v. Mises (1949), p. 663.

Malthusian trap cease to operate?" could be traced back to classical economic theory, which had already deeply influenced philosophy and natural sciences until the middle of the 19th century and whose agenda was not much different from that of current unified growth theory. Over a full century, roughly ranging from 1770–1870, when economics was known as "political economy", demographics played a vital role in the theory of growth. The earlier mercantilist theory, facing regular devastating mortality crises, had viewed a large population as the fundament of (total) national economic prosperity in the international race for scarce resources (see for example Mun 1664). Thereafter, Turgot (1770), witnessing the French population explosion, seems to have been one of the first authors to announce a "law of diminishing returns to labor", according to which a constant production factor (e.g. capital, land) would limit the rise of productivity per person induced by an increase of the labor force. A few years later, Smith (1776) partly revised this physiocratic view in the light of the English Industrial Revolution by stating that high population density and urbanization would cause a greater variety of professions, raising the degree of specialization. If increasingly specialized individuals would reasonably engage in trade, the "division of labor" between these subjects would be enhanced, raising production more than proportionally. Another twenty years later however, the idea that the wealth of nations was based on population growth was struck again when it had become clear that in spite of great technological advances resulting from the division of labor, the English population explosion had effectively pushed down real wages. Malthus (1798) proposed the "principle of population", by stating that population had the inherent tendency to inevitably outgrow production. Another five years later however, Malthus (1803) provided the "great preventive check" as apparently constituting the only justifiable remedy for economies facing excessive population growth and by which individuals were generally susceptible to birth control. Since then, as predicted by the first professor of political economy, fertility abated and productivity increased.

Although the "vaguer intuitions" of the classical economists, as Keynes (1933) put it, provided much deeper and more profound insights than those of modern unified growth theorists, the verbal form of their arguments has at the same time tended to be more favorable to misinterpretations. It is the intention of this work to identify some of those misinterpretations and to partly restore the main ideas of classical growth theory. When Senior (1836) contributed an article to the Encyclopaedia Britannica with the title "An Outline of the Science of Political Economy", he endeavored to summarize the collected scholarly principles of the time, or in other words, the prevailing mainstream theory on economic growth. According to him, there existed common agreement among classical economists with regard to the subsequent four principles.

2.2 The Classical Mechanism of Stagnation

The principle of diminishing returns. It is a well–established fact in neoclassical economic theory that increasing the amount of labor tends to increase overall production. Nevertheless, by holding the stock of all other production factors constant, an incremental amount of labor is generally acknowledged to yield diminishing returns, i.e. to decrease labor productivity. Often referred to as "the principle of diminishing returns to labor" (in the following "Principle of Diminishing Returns" (PoDR)), the mathematical formulation of this effect is displayed by the use of the static neoclassical production function developed by Wicksteed (1894) and Clark (1907) and was popularized by Cobb and Douglas (1928). Accordingly, the PoDR provides a negative "static" causal effect running from labor to productivity. To allow for a clear empirical distinction between the static effect of the PoDR and the dynamic effect of labor division on production, the PoDR will subsequently be greatly simplified. Firstly, as part of a unified model including demographic changes, it will be found useful to replace the term labor with the more general concept "population"(N). Secondly, the negative static causality will be measured using a contemporaneous relationship between GDP per capita(y) and population and is reduced to changes in the denominator of the identity $y_t \equiv Y_t/N_t$, where the time index t refers to the corresponding year. The resulting causal effect might be written as $\partial y_t/\partial N_t < 0$, where a newborn individual will by definition instantly affect GDP per capita. Production (Y) remains unaffected by the PoDR, and population as a production factor will be modeled separately as part of the division of labor.

The principle of labor division. The second principle relates the production factor labor positively to its level of production and comprises the benefits derived from the division of labor. For simplicity, this relationship will be termed the "Principle of Labor Division" (PoLD) and the variable population will again be substituted for pure labor. The effect stemming from the PoLD can be interpreted to correspond to the Kremerian (1993) (or Boserupian 1965) idea by which a larger population raises the chance to discover more productive innovations, although the Smithian principle is less owed to probability, but the logical consequence of a more sophisticated process of specialization. As Young (1928) recalled,

> Senior's positive doctrine is well known, and there were others who made note of the circumstance that with the growth of population and of markets, new opportunities for the division of labour appear and new advantages attach to it. In this way, and in this way only, were the generally commonplace things which they [the classical authors] said about "improvements" [...][10]

However, an increase in population will not yield benefits from the division of labor contemporaneously, but rather lagged. With respect to a newborn individual, the minimum delay to account for a positive increase in production as a response to an increase in population is given by the time span reserved for a basic education, enabling the succeeding generation to participate in the labor market, i.e. to "produce". For simplicity and as it is sufficient to illustrate the role played by the PoLD in the classical framework, only one birth cohort – lagged by one generation – will subsequently be employed in the production function of the form $Y_t = N_{t-g}$.

The above two principles can be formally summarized in the following way.

$$y_t \equiv Y_t/N_t = N_{t-g}/N_t \tag{1}$$

To provide a simple linear relationship, the identity can be approximated using growth rates.

$$\hat{y}_t \approx \hat{Y}_t - \hat{N}_t = \hat{N}_{t-g} - \hat{N}_t = (BR_{t-g} - DR_t) - (BR_t - DR_t) = BR_{t-g} - BR_t \tag{2}$$

where $\hat{N}$, the natural growth rate of population is given by the difference between the birth rate $BR = Births/Population$ and the death rate $DR = Deaths/Population$. Setting $\hat{N}_{t-g} = BR_{t-g} - DR_t$ is justified by the assumption that the death of an average individual is assumed to have an immediate impact on the division of labor, abstracting from infant and child mortality.

Verbally, the principles might be formulated as follows. Firstly, that at the very moment of entering into the economy, every additional individual will statically lower production per capita ($\partial\hat{y}_t/\partial BR_t < 0$). Secondly, that with a delay of at least one generation, total production responds positively, proportionally and indefinitely to an increase in population under the condition that the additional part of the population participates in the division of labor of the economy ($\partial\hat{y}_t/\partial BR_{t-g} > 0$ with g accounting for the generational lag).

[10] Young (1928), p. 35.

The principle of population. Having modeled the impact of population on productivity, the following principle determines the impact of productivity on population. Similar to the PoLD, the third principle is much less utilized in neoclassical models and accounts for the "principle of population" (PoP). Malthus wrote quite unambiguously in his second proposition that

> population invariably increases, where the means of subsistence increase, [...].[11]

Since this work is less concerned with the philosophical argumentation of classical economics and more with the testability of its principles, it is sufficient to assume a positive causal effect of a relative change in productivity (determining the "means of subsistence" per person) on population growth. Again, as the natural population growth rate consists of the difference between birth and death rate, the effect of the PoP might be measured by the effects of changes in productivity growth on both vital rates separately. However, in this preliminary, simple version of a classical growth model, the effects on the death rate will be put back, as wealth effects seem to have played a minor role in the mortality decline and that the fertility decline was the decisive determinant of economic growth.[12] The relationship defining the PoP will subsequently be modeled by a positive effect running from GDP per capita to birth rate. While it is biologically evident that an income effect on birth rate cannot, on average, be realized earlier than nine months after a shock in GDP per capita, and accounting for a lagged fertility decision of not more than one year, it is plausible to suspect fertility to react on average at least one year after the shock took place. Consequently, the following relationship will be employed for simulation: $\partial BR_t / \partial \hat{y}_{t-x} > 0$, with $x = 1$ accounting for the fertility lag.

The first three principles can be interpreted to form the "cycle of misery", which is a sufficient macroeconomic mechanism to account for a model of stagnation. When mathematically formulating these principles, a resulting system of linear equations can be written as

$$\begin{aligned} BR_t &= \alpha_1 BR_{t-1} + \alpha_2 \hat{y}_{t-1} \\ DR_t &= \alpha_3 DR_{t-1} \\ \hat{y}_t &= \alpha_4 BR_{t-15} + \alpha_5 BR_t \end{aligned} \tag{3}$$

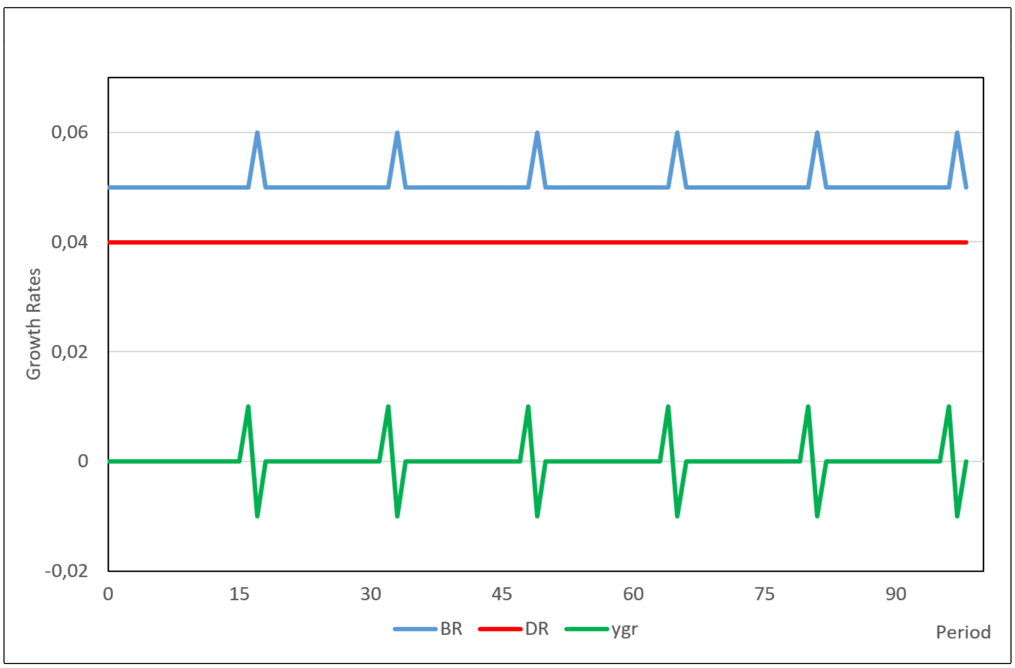
[11] Malthus (1826), book I, ch. I.

[12] Research suggests the following main factors to be responsible for the British mortality decline: The disappearance of the plague (Cipolla (1971)), the introduction of the potato (Nunn and Qian (2011)) and the eradication of smallpox (Davenport et al. (2011)).

where two additional assumptions have been made to arrive at this system. Firstly, the length of one generation is reduced to fifteen years, which seems to be the lowest plausible value. Secondly, since a relatively high persistence is observed for birth and death rates in the model of the demographic transition as opposed to the GDP per capita growth rate, they are assumed to strongly depend on their lagged values. Leaving some room for the interpretation of the relative operation of the principles over time, the magnitude of each effect is represented by an undefined coefficient.

Calibrating the system using $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = -\alpha_5 = 1$, setting initial values $BR_0 = 0.05$, $DR_0 = 0.04$, $\hat{y}_0 = 0.00$ and simulating a one percent shock in $\hat{y}_{15}$ yields figure 2.1, the cycle of misery.

Figure 2.1: A simulation of the mechanism of stagnation.



More explicitly, shocking the growth rate of productivity (ygr) in period fifteen raises the birth rate (BR) one period later owing to the PoP. This increase in population instantly consumes the former gains in productivity due to the PoDR. Hereafter, fifteen periods of stagnation follow until the larger birth cohort has come of age to participate in the labor market, thereby increasing productivity growth via the PoLD, resulting in a further increase of births and so forth. Over time, this short–run mechanism leads to a steady increase in the level of production and population, whereas the growth rates as well as productivity are observed to be relatively stable over the long run. Consequently, the cycle of misery can account for the recorded stylized fact of economic stagnation.

2.3 The Classical Mechanism of Growth

The great preventive check. The last classical principle to be modeled refers to the "great preventive check" (GPC) by which the power of population is repressed from peopling a country fully up to the limits of subsistence. Contrasting the GPC with the PoP, it is advisable to return to Malthus' second proposition in full length:

> Population invariably increases, where the means of subsistence increase, unless prevented by some very obvious and powerful checks.

The checks limiting the natural population growth rate can by definition be exhaustively divided up into those raising the death rate (positive checks) and those reducing the birth rate (preventive checks). As the positive checks "moral and physical evil" — which may be briefly summarized as war, epidemics and famine — are supposed to be non–existent after having completed the mortality decline, what are the "obvious and powerful" preventive checks that are suggested as being capable of reducing the rate of population growth within manageable limits? Malthus referred to the GPC as "prudential restraint from marriage".[13] Accordingly, every individual faces the choice between reproduction ("marriage") and the preservation of its social rank during the early stages of its life. Further inquiries have shown that reproduction is in most cases not accomplished until a certain social rank has been achieved (see for example McCulloch 1863). However, after a general increase in life expectancy (corresponding to the decline in mortality), a higher social rank cannot be achieved until the later part of life, postponing reproduction until the individual's average biological fertility interval has often been exceeded. This interpretation of the fourth principle is confirmed by Malthus' conclusion that

> it will be generally found true, that the increasing healthiness of a country will not only diminish the proportions of deaths, but the proportions of births and marriages.[14]

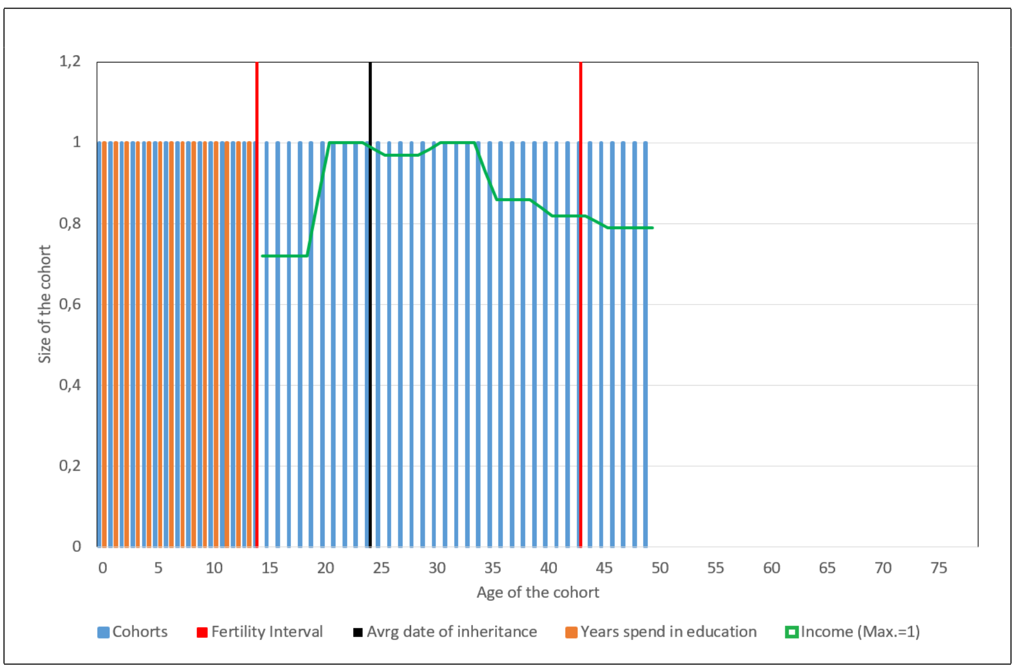
[13] Senior (1836), p. 143: "Our readers are of course aware that, by the word "marriage," we mean to express not the peculiar and permanent connection which alone, in a Christian Country, is entitled to that name, but any agreement between a man and woman to cohabit under circumstances likely to occasion the birth of progeny."

[14] Malthus (1826), book III, ch. II.

As a consequence, the birth rate is positively causally determined by the death rate and the operation of the GPC is modeled by $\partial BR_t/\partial DR_{t-x} > 0$, again delayed by the cumulative lag of pregnancy and fertility decision.

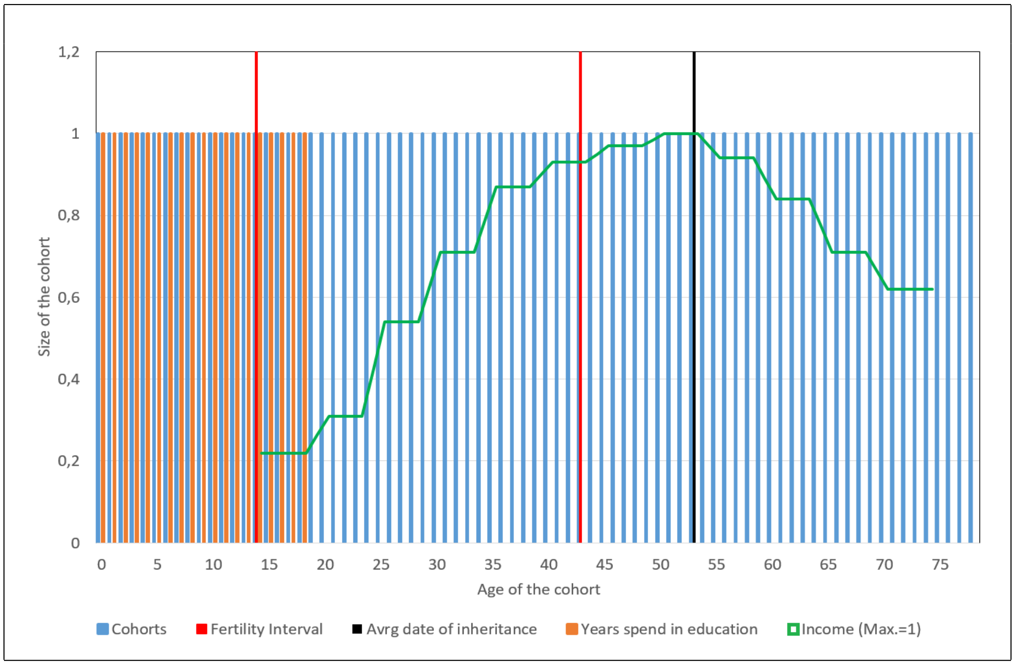
Nevertheless, a more precise mathematical formulation of classical growth theory requires the GPC to be further analyzed to clearly distinguish between the particular effects of mortality on fertility. For, on the one hand, there exist mortality effects that directly act on fertility, notably an "inheritance effect" and an "infant mortality effect", while on the other hand, mortality effects operate indirectly through the income channel, weakening the PoP. The latter will be named "average income effect" and "sexual selection effect".

Figure 2.2: Stylized population structure of England in 1830. Source: See Burnette (2006) for income statistics.



To trace the evolution of those effects, figures 2.2 and 2.3 represent the stylized population structures for the years 1830 and 2010 respectively. For ease of illustration, populations are assumed to be stationary and stable, i.e. the birth rate equals the death rate and its relative age distribution does not change over time. The resulting cylindrical rather than pyramid form implies that every individual dies at the age of its life expectancy.[15] Average life expectancy can be recovered from the inverted death rate, which was roughly 0.02 in 1830 and 0.0125 in 2010, excluding infant mortality.

Figure 2.3: Stylized population structure of England in 2010. Source: Bureau, U.C. (2011) for income statistics.



The average income effect. To begin with the stylized population structure in 1830, individuals lived for 50 years on average, with the first fifteen years spent on education. The fertility interval is taken to be constant, ranging from 15–45 years. As a result, 86% of the working population (benefiting from increases in income) was fertile, whereas in 2010, when life expectancy was roughly 80 years, only 42% of the working population was capable of reproduction (see blue shaded area). Accordingly, positive GDP per capita growth was in the latter situation increasingly distributed to infertile individuals of high age, who were not even able to convert the additional income into children. It is quite obvious that, if wealth is mainly distributed to an infertile population, Malthus' notion that "population invariably increases where the means of subsistence increase" ceases to be true. This shift in social fertility is the first effect that can account for a breakout from the cycle of misery.

The sexual selection effect. Furthermore, it can be observed that the life period during which the average individual earned its maximum income (green line) shifted

[15] The effect of early mortality is dealt with as part of the infant mortality effect .

from the young age of 20–35 years in 1830 to the old age of 45–60 years in 2010. As it is well–known that individuals' choices on their partners are in a high degree positively affected by the latter's social rank, and as the individuals' social rank is quite reliably reflected by its relative level of income, it is a logical inference to presume a postponement of marriages between 1830 and 2010, resulting in an increasingly delayed fertility decision.

The inheritance effect. Thirdly, the birth rate is directly affected by the death rate of those individuals who possess a part of the economy's wealth. With the death of such an individual, its possession is usually bequeathed to the succeeding generation. Since the age of women at their first birth was approximately 25 years in 1830 and has not changed drastically over the last two hundred years and since their husbands are currently, quite similar to 1830, on average merely three years older, inheritance is quite universally passed to the offspring some 25 years before their own deaths.[16] Consequently, average age of inheritance was approximately 25 years in 1830 and around 55 years in 2010 (see black bar). Since early inheritance formerly allowed individuals to take over and make use of their parents' capital, often in form of a business, it tended to greatly increase their income and social rank, favoring "early marriage" and subsequently conversion of wealth into progeny. Until 2010 however, the channel for translating inherited wealth into a higher number of offspring was increasingly closed down, as the heir will with a high probability have arrived at an infertile age.

The infant mortality effect. Complementing the above impact of the death of an old individual on fertility, the early death of individuals at a very young age provides another well–known direct reason for high birth rates, completing the generation conflict. The diminution of infant and child mortality in the aftermath of the epidemiological transition seems to have induced parents to dispose of some formerly necessary replacement births (See for example Haines 1998). Over time, this effect eased the social pressure on individuals to marry early, further postponing reproduction.

Summing up the outcome of these four effects of mortality on fertility, it might be stated that if two succeeding generations exist at the same time, a further rising life expectancy will progressively cause a generation conflict, forcing young individuals to preventively check their fertility.

As should have become clear by now, in classical theory the great preventive check accounts for the missing link between the mechanism of stagnation and the mechanism

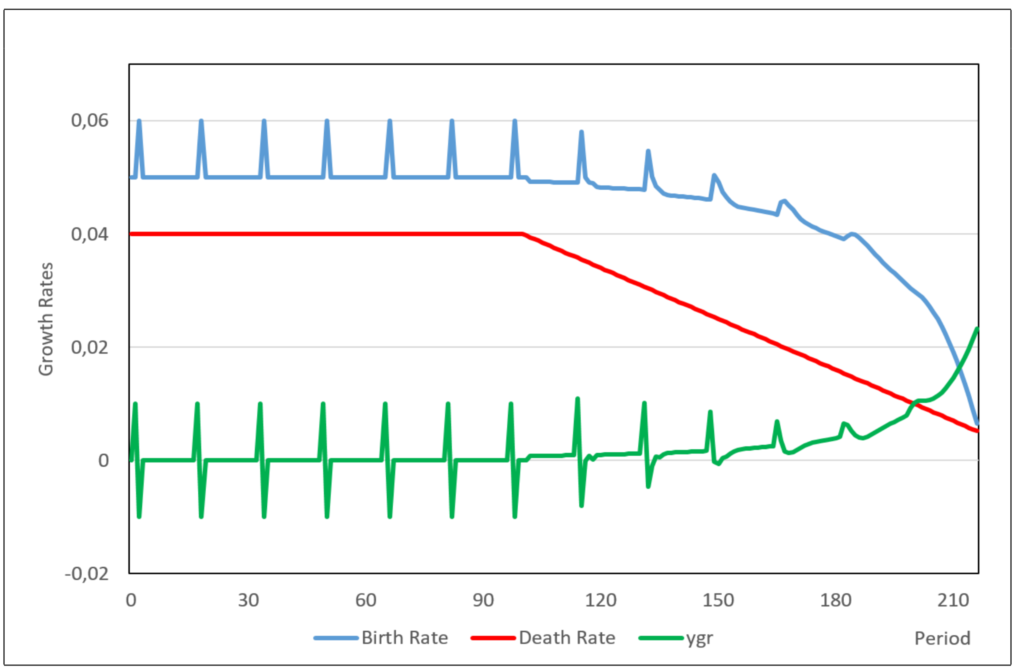
[16] See for example Hajnal (1965) or Clark (2007) for historical marriage pattern.

of growth and was intended by Malthus to solve the economic problem. The direct mortality effects on fertility and the indirect effects operating through the income channel are incorporated into the mechanism of stagnation by employing the following system of equations.

$$\begin{aligned} BR_t &= \alpha_1 BR_{t-1} + \overbrace{\alpha_6(DR_{t-1} - DR_{t-2}) + \alpha_2}^{Great\,preventive\,Check} \overbrace{\hat{y}_{t-1}}^{PoP} \\ DR_t &= \alpha_3 DR_{t-1} - \alpha_7 t \\ \hat{y}_t &= \underbrace{\alpha_4 BR_{t-15}}_{PoLD} + \underbrace{\alpha_5 BR_t}_{PodR} \end{aligned} \tag{4}$$

where the coefficients $\alpha_2 = 10 * DR_{t-1}$ and $\alpha_6 = \alpha_2^{-1}$ account for the indirect and direct mortality effects and vary with the level of the death rate. On the one hand, the GPC is induced by an increasing rise of the mortality effects, directly reducing the birth rate. On the other hand, the GPC indirectly reduces fertility by mitigating the positive income effects. The system is supplemented by a negative linear trend in the death rate, reflecting exogeneity of the mortality decline. For calibration, coefficients and initial values from the former section are retained and α_7 is set to the value .0003.

Figure 2.4: A simulation of the mechanism of growth.



The results from the simulation are displayed in figure 2.4. The first 100 periods of the simulation correspond to the evolution of the regime of stagnation as it has been modeled before, following a shock in $\hat{y}_t$. The second part accounts for the evolution of the regime of growth and is triggered by the linear decline in death rates. This decline decisively induces the progressive operation of the great preventive check according to the first equation of (4). Owing to the direct mortality effect, the birth rate eventually declines even more rapidly than the death rate. In the case of the indirect effects, the short–run conversion of productivity into births owing to the PoP decreases in magnitude. Put differently, the potential for economic growth is triggered by the fact that birth cohort size decreases over time. If the ratio BR_t/BR_{t-15} was larger than one, the negative effect of diminishing returns due to an ever–growing population outweighed the positive long–run effect of the birth rate on labor division. However, as long as the ratio BR_t/BR_{t-15} decreases, i.e. the birth rate declines over the course of one generation as is observed in figure 2.4, the ratio between unproductive and productive individuals abates as well. In this case, the productivity gains from labor division outperform the losses from diminishing returns, resulting in the observed stylized facts. This simulation affords a confirmation of the modeled mechanisms of stagnation and growth to match the stylized facts, furnishing classical growth theory with a consistent mathematical framework.

3 Empirical Evaluation of the Classical Mechanisms

3.1 Empirical Model

In analyzing whether the empirical relationships match the above interpretation of classical growth theory, the author regards time series analysis as being the most appropriate tool. Since the times of Malthus, economists have tried to make sense of the apparent link between demographic and economic variables observed in the stylized facts. Only recently however, with the construction of Wrigley and Schofield's (1981) preindustrial time series on birth rates and death rates, quantitative studies were able to state definite evidence of falsifiable hypotheses. While Lee's (1981) methodology employed distributed lag regressions, Eckstein et al. (1986) attempted to test their hypotheses using a vectorautoregression (VAR). Nicolini (2007) refined this approach by illustrating impulse response functions that allow for comparability of effects between the variables across economies and over time. Building on the VAR and developing a more sophisticated methodology, Herzer et al. (2012) employed a VEC model to account for possible cointegration between the variables, while Rathke and Sarferaz (2014) introduced time-varying coefficients.

This paper will retain the traditional VAR approach for the following reason. While the above estimations were usually based on the usage of a level variable of real wages or real GDP per capita, they will in this case be replaced with growth rates of real GDP per capita, which is justified as follows. Firstly, the major part of the true relationships between the variables becomes linear only when employing growth rates, and a linear relationship is required to apply a simple OLS estimation. Secondly, as growth rates display the same unit of measurement across economies, cross-country data could be used to assess international comparisons. Thirdly, instead of level variables, growth rates are most arguably stationary, which is required to avoid spurious autoregressions when not accounting for cointegration.

3.1.1 Vectorautoregression

To evaluate the hypotheses in question, the statistician faces the problem of endogeneity between the variables birth rate, death rate and GDP per capita growth. Eckstein et al. (1986) suggested a VAR model as being capable of solving this problem by treating all variables as endogenous. Initially, the system constructed in the last chapter might be written in matrix notation as

$$\begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_6 & \alpha_2 \\ 0 & \alpha_3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & \alpha_6 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-2} \\ y_{t-2} \\ z_{t-2} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-15} \\ y_{t-15} \\ z_{t-15} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_5 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha_7 \\ 0 \end{pmatrix} t. \quad (5)$$

The idea of the VAR approach is to recover the relevant coefficients from an OLS regression of contemporary values on lagged values of the variables and to use the recorded parameters to project the average impact of an exogenous shock in one of the variables. The obtained impulse response functions are expected to conform to the classical principles as formulated in chapter three. However, for the linear system to qualify as a VAR representation, some further reservations will be made in the following.

3.1.2 Stationarity of the Variables

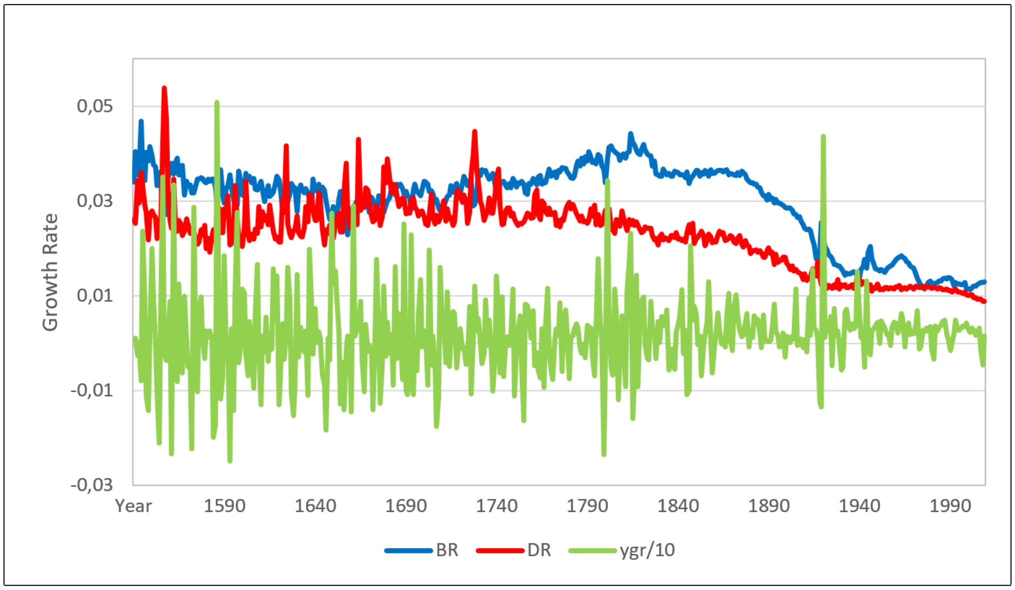
An OLS estimation over time requires at least some of the single data series to be stationary, as integrated or trended variables will almost certainly give spurious results. Since the English and Welsh data[17] provide the longest national time series available, ranging from the year 1541 to 2010, tests on the order of integration as well as the tests for lag selection will be representatively conducted on this sample. The annual data on which the VAR model will be based are displayed in figure 3.1.[18]

In the case of GDP per capita growth, the results from running Augmented Dickey–Fuller tests on non–stationarity seem to unequivocally indicate stationarity of the variable, while the application of the same test to death rate and particularly to birth rate does not always reject the null hypothesis of non–stationarity on a 1% level (see table 3.1). Indeed, the pattern of birth rate and death rate has led to an extensive debate on their order of integration.

Firstly, following Nicolini (2007), vital rates could be treated as stationary variables, as it seems implausible to believe that they have ever exceeded a certain maximum value, say ten percent, or that they have fallen below a minimum value, say zero, in the long run. Despite vital rates displaying high persistence, they may generally be assumed to be stationary, as their values represent (population) growth rates and are by definition restricted to lie within the range (0,1). Accordingly, they cannot in reality follow a random walk or a trend and the assumption $0 < \alpha_1, \alpha_3 < 1$ should hold. Nevertheless, stationarity of these two variables might be questioned by having found evidence of the variable natural population growth rate being stationary on a 1% level (see table 3.1).

[17] In the following referred to as the English data.
[18] GDP per capita growth is divided by ten for better visualization.

Figure 3.1: England & Wales: Time series on birth rate, death rate, GDP per capita growth 1541–2010. Sources: Clark (2009), Mitchell (2013), Wrigley and Schofield (1981).



As the latter is by definition a linear combination of birth rate and death rate, there is strong indication for the vital rates being cointegrated (see e.g. Herzer et al. (2012)). However, as was pointed out by (Fanchon and Wendel (2006)), "VAR models can be estimated with data on stationary and non-stationary variables if the non-stationary data is also cointegrated because recent theoretical work proves that estimation with such data will yield consistent parameter estimates."[19] Thirdly, as was suggested by

Table 3.1: England & Wales: Unit root tests on the relevant variables.

Augmented Dickey–Fuller test for unit root			Number of obs = 468		
model	1% Crit. Value	Test Stat. GDP pc gr	Test Stat. Birth Rate	Test Stat. Death Rate	Test Stat. Pop growth
2 lags, no constant	-2.580	-16.737	-1.235	-1.451	-3.747
2 lags, constant	-3.443	-17.460	-0.941	-3.539	-6.303
2 lags, linear trend	-3.981	-17.620	-1.682	-5.925	-6.304

[19] Using a VEC model specification similar to that of Herzer et al. (2012), accounting for the potentially integrated variables birth rate and death rate, or estimating a restricted model like that of eq. (5) do not yield very different results.

Sims (1980), if we are rather interested in the nature of relationships between variables with the end purpose being estimation of the impulse response functions to capture the dynamic responses and less interested in point estimates, estimating a VAR with non-stationary variables can give us important insights on short-run relationships. Accordingly, the question of the order of integration of the vital rates does not pose problems with regard to a consistent estimation of a VAR model.

3.1.3 Ordering of the Variables

The estimation of an unrestricted VAR(3) model of the above form is complicated by the inclusion of contemporaneous effects, required to measure the PoDR. To analyze the interactions between annual demographic and economic variables, Nicolini (2007) proposed a recursive VAR structure based on Theil (1971) of the vector form

$$A_0 Y_t = \textstyle\sum_{j=1}^{s} A_j Y_{t-j} + u_t \tag{6}$$

where the vector Y_t contains the contemporary values of the endogenous variables, each of which depends on its own lagged values and on contemporaneous and lagged values of the other variables. A_j are the coefficient matrices of the lagged values. The components of the residuals u_t are supposed to be uncorrelated, i.e. "clean" of those contemporaneous effects that are already included in the coefficient matrix A_0 ("orthogonalized residuals"). Multiplying both sides by A_0^{-1} yields the conventional VAR form

$$Y_t = \textstyle\sum_{j=1}^{s} (A_0^{-1} A_j) Y_{t-j} + (A_0^{-1} u_t) \, with \, E(u_t u_\tau^{'}) = \begin{cases} I \; if \; t = \tau \\ 0 \, otherwise \end{cases} \tag{7}$$

that might be rewritten as

$$Y_t = \textstyle\sum_{j=1}^{s} \Phi_j Y_{t-j} + \epsilon_t \, with \, E(\epsilon_t \epsilon_\tau^{'}) = \begin{cases} \Sigma \; if \; t = \tau \\ 0 \, otherwise \end{cases} \tag{8}$$

where consistent estimators of Σ and the Φ_j's are easily obtained by running OLS regressions equation by equation.

Additionally, estimation of A_0^{-1} is necessary to recover the response of the variables to orthogonalized shocks. However, as this requires estimation of an additional number of parameters, the system is not identified. A sufficient condition to reduce the amount of parameters is to restrict the VAR model by imposing lower triangularity of the matrix A_0^{-1} from using a Cholesky decomposition $\Sigma = A_0^{-1} A_0^{-1'}$. Multiplying the residuals by a lower triangular matrix implies that, given a particular ordering inside the vector Y_t, each variable is allowed to react within the current period to a shock in any of the variables of a higher ordering, while it is completely unresponsive to shocks in variables that are lower in the ordering. In this context, yearly demographic variables seem to fit the framework almost ideally as it can be clearly distinguished between contemporaneous and lagged effects. In the last chapter it was concluded that childbirth rarely takes place in the same year as the fertility decision, in particular due to a pregnancy lag. Since this natural lag prevents it from being contemporaneously effected by death rate and GDP per capita, birth rate is the only plausible candidate to be the first variable in the vector Y_t. Furthermore, the death rate is placed as second variable to preserve the possibility of contemporaneous effects on GDP per capita due to the PoDR and the PoLD, which have so far been assumed to neutralize each other. As a consequence, it is assumed that a change in GDP per capita does not affect the death rate in the same year, while a delayed negative effect retains the possibility of an endogenized mortality, yielding the following system to be estimated:

$$\begin{pmatrix} BR_t \\ DR_t \\ \hat{y}_t \end{pmatrix} = \Phi_1 \begin{pmatrix} BR_{t-1} \\ DR_{t-1} \\ \hat{y}_{t-1} \end{pmatrix} + \Phi_2 \begin{pmatrix} BR_{t-2} \\ DR_{t-2} \\ \hat{y}_{t-2} \end{pmatrix} + \Phi_{15} \begin{pmatrix} BR_{t-15} \\ DR_{t-15} \\ \hat{y}_{t-15} \end{pmatrix} + \begin{pmatrix} \beta_1 & 0 & 0 \\ \beta_2 & \beta_3 & 0 \\ \beta_4 & \beta_5 & \beta_6 \end{pmatrix} \begin{pmatrix} u_{BR_t} \\ u_{DR_t} \\ u_{\hat{y}_t} \end{pmatrix} \tag{9}$$

The exogenous trend employed in the simulation is supposed to be captured by Φ_1.

3.1.4 Lag Order Selection

In the foregoing simulation, the benefits from the division of labor were strongly simplified. However, there are at least two important reasons complicating their measurement in empirical analyses. Firstly, since national data are used without accounting for the international labor division, the effects of foreign population growth on domestic output are not captured in the regression. Since external trade shocks might be suspected to cause a major part of the strong fluctuations of GDP per capita data as shown in figure 3.1, this effect should not be underestimated. Secondly, to roughly illustrate the positive delayed effect of births on the labor market, a lag of fifteen years was employed in the simulation. For all real applications, the exact timing of an average individual entering the division of labor cannot be sufficiently determined, much less the resulting benefits. Accordingly, it is assumed that a VAR model is too "costly" in terms of parameters to be able to significantly estimate the effect of the PoLD after one generation and the fifteenth lag will be eliminated from estimation. This issue will be dealt with in future research. On the other hand, omission of the fifteenth lag increases the number of degrees of freedom, which is valuable when using small sample sizes. It would nevertheless be advisable to include a third lag by which some additional information regarding the PoLD, stored in the remaining error terms, might be captured. The use of a VAR(3) model is supported by running a series of lag–selection tests on the English data, as the most parsimonious model is suggested by the Schwarz–Bayesian information criterion to use three lags (see table 4.1 in app. I). With regard to a delayed fertility decision, a lag of three years appears plausible as well, whereas every additional lag may unnecessarily increase the number of parameters to estimate. Replacement of the fifteenth by the third lag gives

$$\begin{pmatrix} BR_t \\ DR_t \\ \hat{y}_t \end{pmatrix} = \Phi_1 \begin{pmatrix} BR_{t-1} \\ DR_{t-1} \\ \hat{y}_{t-1} \end{pmatrix} + \Phi_2 \begin{pmatrix} BR_{t-2} \\ DR_{t-2} \\ \hat{y}_{t-2} \end{pmatrix} + \Phi_3 \begin{pmatrix} BR_{t-3} \\ DR_{t-3} \\ \hat{y}_{t-3} \end{pmatrix} + \begin{pmatrix} \beta_1 & 0 & 0 \\ \beta_2 & \beta_3 & 0 \\ \beta_4 & \beta_5 & \beta_6 \end{pmatrix} \begin{pmatrix} u_{BR_t} \\ u_{DR_t} \\ u_{\hat{y}_t} \end{pmatrix}. \tag{10}$$

3.1.5 Impulse Response Analysis

To find evidence for the classical growth model, the suggested linear relations should be approximately recovered by applying impulse response analysis to the above restricted VAR(3) model. To this end, nine orthogonalized impulse response functions are computed by shocking the error terms of each variables' equation by one standard deviation. The initial shock instantly affects the assigned contemporaneous variables and subsequently propagates through the system. Since childbirth is, as a response to shocks in death rate and GDP per capita growth, most arguably spread over a number of years, it is reasonable to expect *accumulated* orthogonalized impulse response functions to yield a more pronounced effect. On the other hand, as the period in question should not exceed the short term, a time horizon of more than five years seems inappropriate granting that the fertility decision is usually made after four periods and that a longer horizon will not provide additional information. If the considerations made in chapter three are correct, the causalities given by the estimated cumulative orthogonalized impulse response functions (coirfs) following a shock in u should be of the form

$$\begin{pmatrix} BR \\ DR \\ \hat{y} \end{pmatrix} = \begin{pmatrix} high\,persistence\,(+) & short\,run\,(+)^1 & short\,run\,(+)^2 \\ (x) & high\,persistence\,(+) & (x) \\ contempor.(-)^3, long\,run\,(+)^4 & (x) & low\,persistence\,(+) \end{pmatrix} \begin{pmatrix} u_{BR} \\ u_{DR} \\ u_{\hat{y}} \end{pmatrix} \tag{11}$$

where $(+)^1$ is expected to display the positive average effect of the GPC and $(+)^2$ to capture the positive average effects of the PoP. $(-)^3$ is supposed to represent the negative effect of the PoDR. This relation exists by definition and the effect will be observed as long as it is not outweighed by the impact of the PoLD. As was mentioned, $(+)^4$ will not be captured sufficiently well to account for the positive effect of the PoLD and its presence even poses a threat to a clear identification of the PoDR. However, although PoDR and PoLD may not be clearly identifiable within the VAR(3) framework, their existence can certainly not be denied. Consequently, the subsequent investigation will chiefly evaluate the hypotheses of the GPC and the PoP. Persistence effects are expected to be measured for the variables birth rate and death rate, much less for GDP per capita growth. The remaining three impulse response functions denoted (x) will also be estimated to capture further potential effects by which the classical model might be extended ex post. To test for significant effects, 200 bootstrap replications are used to generate 95% confidence intervals. The resulting impulse responses should be interpreted with caution, as the effects of the GPC and the PoP are supposed to be time–varying in magnitude, whereas the estimation can merely give average results over the whole period in question.

3.1.6 The Data

The above illustrated time series are taken from Wrigley and Schofield (1981) for vital rates until 1870, Mitchell (2013) for vital rates and real GDP per capita after 1870 and Clark (2009) for real GDP per capita. In search of a unified theory formed by universal principles, it is essential to investigate global data. In this case, Mitchell's International Historical Statistics arguably provide the longest and most comprehensive official global series on vital rates and GDP per capita. The database was partly corrected by the author to eliminate some obvious typing errors.[20]

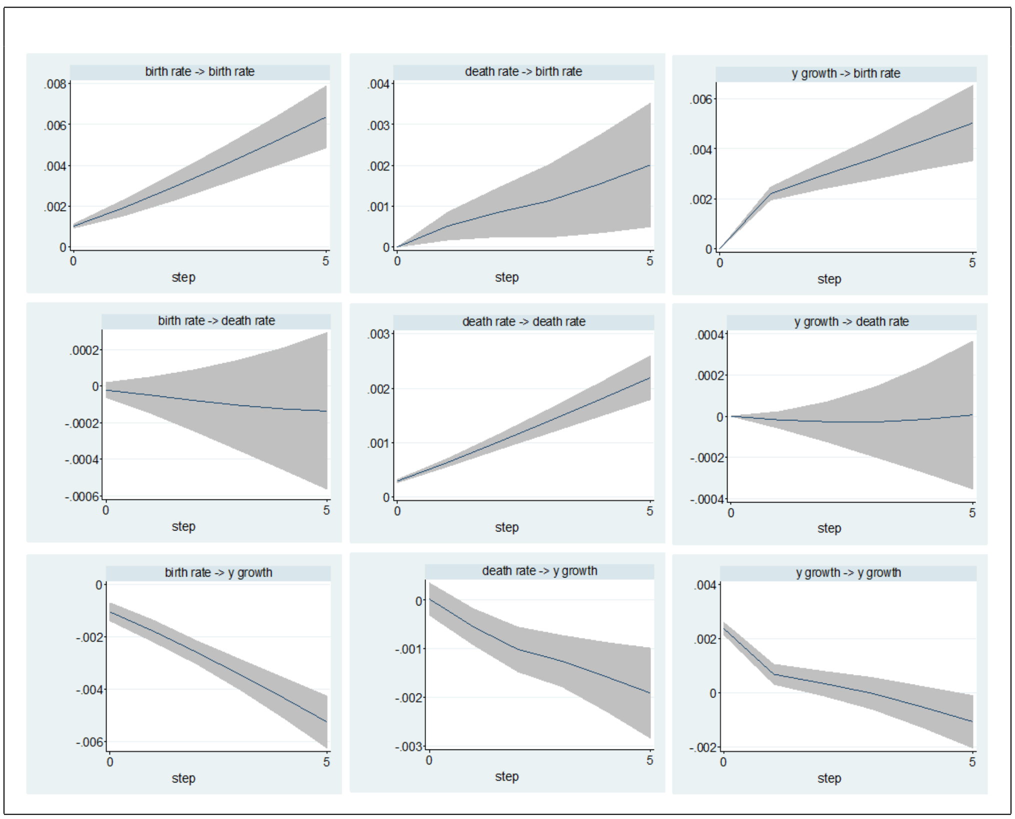
[20] The adjusted data are available upon request.

3.2 Estimation Results

3.2.1 Simulation

As a very useful reference point, it is advisable to first run the above VAR(3) estimation on the simulation given by figure 2.4, i.e. eq. (5) using the corresponding calibration from chapter 2.[21] The coirfs resulting from this estimation are expected to deliver a benchmark against which the ensuing real samples might be compared. The size of the shocks is given by the standard deviation of the corresponding variables. The universal average effect of the GPC seems well exposed in the upper central graph of figure 3.2.

Figure 3.2: Simulating the classical growth model: Coirf matrix based on a VAR(3) model.



Likewise, the universal average effect of the PoP appears quite nicely depicted in the upper right graph. Both effects are statistically significant, with the GPC on a 5%-level and the PoP on a 1%-level. As suggested, the positive lagged effect of the PoLD of birth rate on GDP per capita growth after one generation cannot be captured

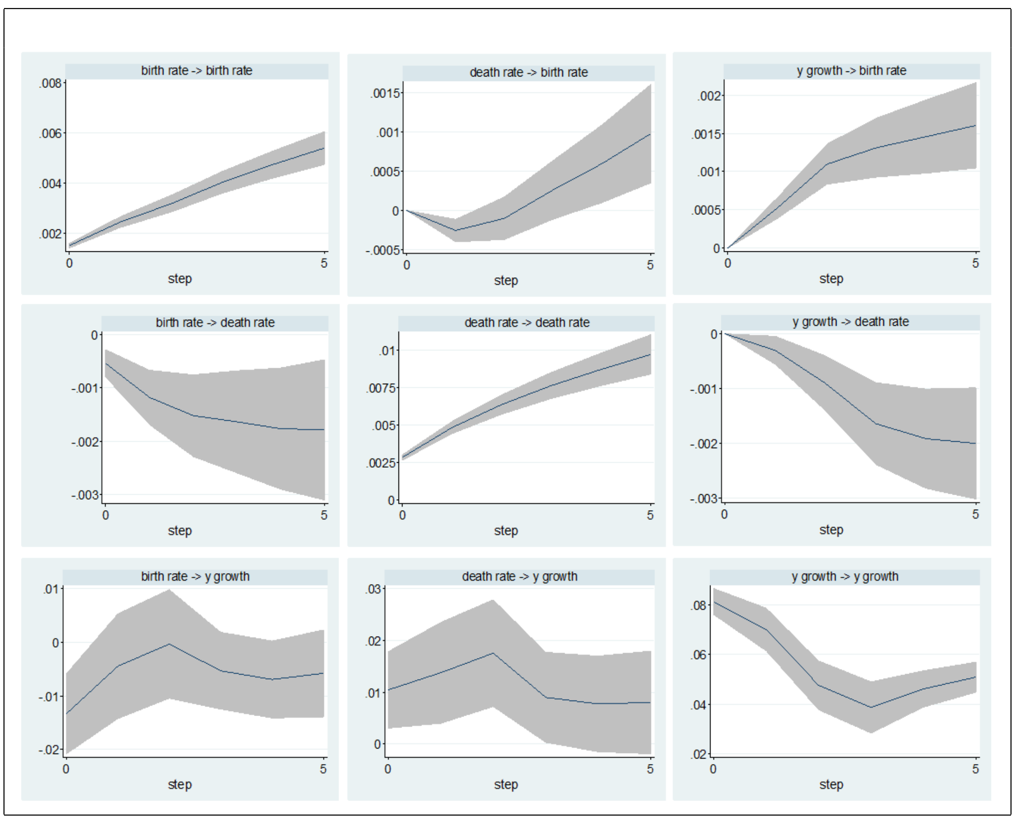
[21] To provide some additional variation on the variable death rate, the right hand side of the second difference equation of eq. (4) is supplemented by adding an annual shock $\epsilon_t \sim U(-0.005, 0.005)$.

with a maximum lag length of three. Instead, the undistorted contemporaneus annual effect of the PoDR is significantly displayed in the bottom left graph.

3.2.2 England and Wales 1541–2010

The computed coirfs from running the VAR(3) model on the English data are displayed in Figure 3.3. The effect stemming from the PoP is roughly in line with that of the simulated model. The reaction of birth rate to a shock in GDP per capita growth is positive and significant on a 1%-level in the first period already, indicating a quick fertility adjustment, and accumulates in magnitude over the subsequent periods. With regard to the GPC, a death rate shock does not induce birth rate to react after one period, pointing at a lagged fertility decision. After four years however, the positive effect becomes statistically significant on a 5%-level, providing evidence of a positive causal relationship.[22]

Figure 3.3: England & Wales 1541–2010: Coirf matrix based on a VAR(3) model.



[22] In the simulation of chapter three, a lag of one year was used. However, this assumption can be easily replaced by a fertility lag of up to four years.

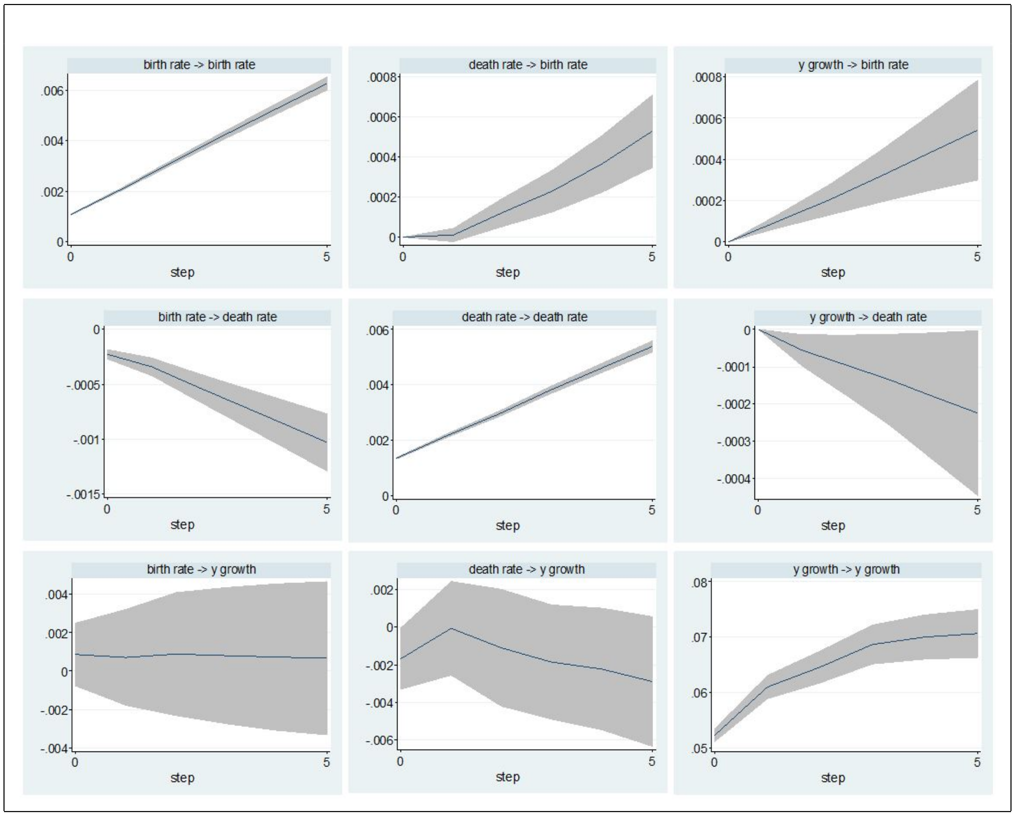
Although the effect of the PoDR is significantly measured in year zero after a shock in birth rate, the subsequently displayed response until year five might have been derived from the PoLD. While in the simulated impulse response the negative contemporaneous effect of the PoDR seems to have accumulated over the subsequent periods , this accumulation is neutralized in the English sample, suggesting that a (possibly long-run) positive effect of birth rate on GDP per capita growth has been captured by additional residual correlation. Hence, the hitherto observed coirfs indicate strong support for the classical causalities in the English case.

3.2.3 Stacked Model of 90 Economies

Mitchell's International Historical Statistics provide data on vital rates and GDP per capita for 94 nations. Out of these, 90 nations exhibit simultaneous data on the three variables over at least three consecutive years. However, with an average number of approximately 44 observations, the impulse responses of those 90 eligible countries can, when individually tested, not be expected to give sufficiently reliable evidence of the classical model. However, if they could be computed collectively, the number of observations would rise to 3,952. For that purpose, the individual country–level data are stacked into one sample, leaving space for three "missing values" between sub–samples such that the last observation of the preceding and the first observation of the succeeding country are not related to each other.

On the one hand, the resulting coirfs displayed in Figure 3.4 match those of Figure 3.3 relatively well. On the other hand, as the stacked sample includes observations between the years 1815 and 2010 only, while the English sample ranges from the year 1541 to 2010, the former is suspected to mainly include information on the growth regime. It is therefore not surprising to observe weaker persistence effects in all three variables when accounting for a regime of stagnation in the English case. Furthermore, the effect of the GPC is remarkably pronounced compared to the English model, while that of the PoP is smaller in magnitude, suggesting time-varying effects. However, as in the English sample, it remains unclear why the fertility decision seems to be lagged by an additional year in the case of the GPC as compared to the effect of the PoP. Again, the time-invariant accumulated effect of the PoDR seems to be neutralized by the time-invariant PoLD such that no significant effects can be observed. In any case, both analyses record qualitative and quantitative evidence of the GPC and the PoP.

Figure 3.4: 19 Economies 1815–2010: Coirf matrix based on a VAR(3) model.



3.2.4 Robustness of the Estimation on a Country-level

In the following, the robustness of the GPC and the PoP will be evaluated on a country-level. Since the number of parameters of the above VAR(3) model amounts to 36 when including a vector of intercepts, it does not seem reasonable to evaluate economies with less than 70 observations on death rate, birth rate and GDP per capita growth rate. Accordingly, all 19 available country–level time series providing at least 70 observations on the three variables are employed for empirical evaluation. (see table 4.2 in app. I for the corresponding countries) Following the above estimation procedure, it should be kept in mind that the resulting coirfs are naturally suspected to be less significant due to the smaller sample sizes.

The correspondingly estimated coefficients of the GPC and the PoP after four years are displayed in table 3.2, including the English case. For a more detailed examination of the stability of the effects, the complete national coirfs are displayed in figures 4.5–4.8 in app. II. Astonishingly, in spite of the small sample sizes, with the exception of Denmark

Table 3.2: Cumulative orthogonalized impulse response of BR in % 4 periods after a one standard deviation shock in DR and g_y. All 20 countries with more than 70 observations are displayed.

country	Principle of Population $g_y \rightarrow BR(4)$	Great preventive Check $DR \rightarrow BR(4)$	obs
arg	0.039	0.018	97
au	0.018	0.077**	75
aus	0.035	0.115***	133
can	0.052	0.039	100
chil	0.097	0.064	99
den	-0.027	0.075***	179
fin	0.051	0.172***	147
fra	0.076***	0.059***	182
ger	0.148***	0.089***	122
hun	0.161***	0.038	72
ita	0.057	0.035	102
jap	0.005	0.066*	109
net	0.015	0.067**	91
nor	0.008	0.077***	128
nz	0.013	0.024	76
rom	0.034	0.043	79
spa	0.074**	0.003	91
swe	0.053	0.108***	146
swi	0.017	0.047**	78
e&w	0.146***	0.0596**	467

*** indicates significance at 1% level, ** at 5% level, * at 10% level.

each of the 20 economies display a positive value in the case of the GPC as well as for the effect of the PoP. Additionally, 17 out of 40 tests display significant coefficients, suggesting that even very small samples are capable of providing evidence for the two universally operating principles and sustaining the results of the stacked model.

3.2.5 Time-varying effects

When accounting for the time-varying effects modeled in the first difference equation of eq. (4), classical theory suggests that the GPC grows stronger whereas the PoP grows weaker during the transition to growth. Again following Nicolini (2007), a straightforward way to measure their evolution in the form of average mortality and income effects, which are supposed to increasingly respond to the declining level of the death rate, is to split up the English sample into an early period of economic stagnation and high mortality and a late period of economic growth and low mortality and to compare the respective coirfs. As, in accordance with the stylized facts, the growth take-off corresponds to the fertility decline, 1815 is chosen as the cutoff year, as it exhibits the maximum value and a structural break for birth rates. However, with the first sample employing 271 observations and the second sample using 192 observations only, the outcome can merely be considered as indicative evidence.

Figure 3.5: England & Wales: The evolution of GPC, PoP, PoDR and "positive checks". Upper sample 1541–1815. Lower sample 1815–2010.

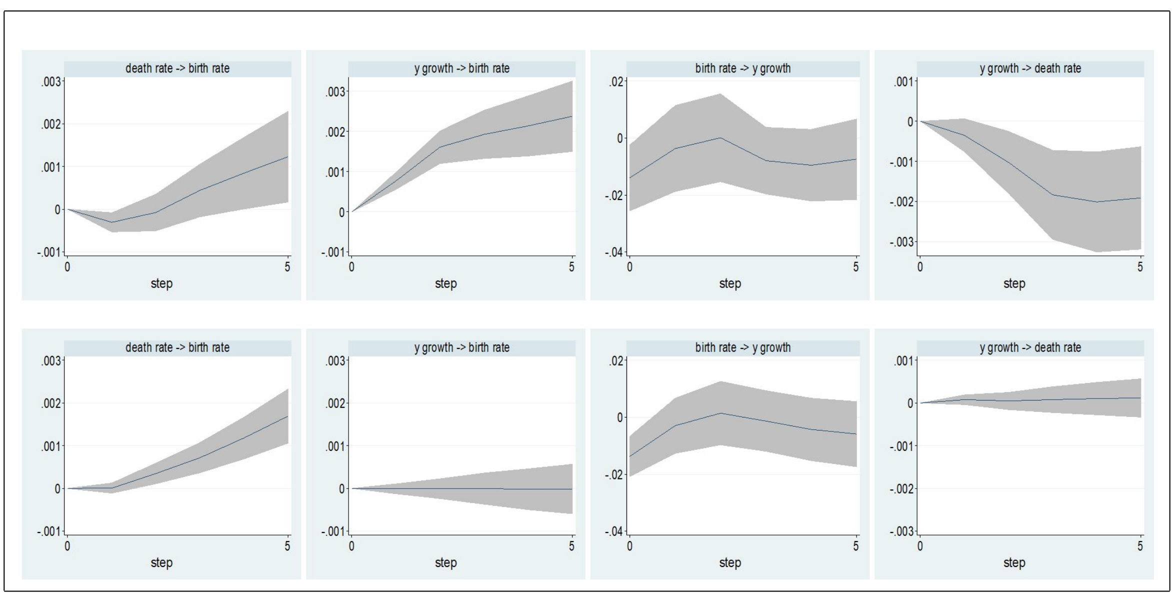


Figure 3.5 illustrates the evolution of the GPC, the PoP and PoDR respectively from left to right. In the upper row, coirfs are given for the timespan 1541-1815, the bottom row displays responses for the period 1815-2010. First, the effect attributed to the GPC grows in size and significance over time, suggesting an increasing direct mortality effect as is predicted from theory. Secondly, the impact of the PoP sharply decreases over time. While indicating a strongly positive conversion rate of GDP per capita growth into birth rate in the early sample, the effect seems to entirely disappear in the later period. This diminishing income effect is in line with the increasing operation of indirect mortality effects. However, as it seems implausible to argue that the effect of the PoP completely vanished after 1815, the extremely low values of the coirf might indicate distortions resulting from the small sample size. Thirdly, the relatively unchanged coirf that is supposed to capture the constant contemporaneous effect of the PoDR and the time-invariant effect of the PoLD substantiates the theoretical predictions and supports robustness of the estimation method.

3.2.6 A Critical Note on the Prevailing Measurement of Preventive and Positive Checks

Lastly, having found evidence of the existence of the short–run mechanisms suggested by classical growth theory, an additional effect will be briefly interpreted, as it is regularly used in the prevailing empirical literature. The effect that might be important to consider arises from the statistically significant negative lagged response of death rate to changes in GDP per capita growth as is illustrated by the respective central right coirfs of figures 3.3 and 3.4. So far, exogeneity of the death rate has been assumed to trigger the epidemiological transition. As a short–term relation, however, GDP per capita growth seems to have affected mortality even before the epidemiological transition, since this effect can be found to prevail in the early English data sample (see figure 3.5, upper right graph) and to wear off in the late sample (see figure 3.5, lower right graph).

Some authors (Nicolini 2007; Crafts and Mills 2009; Pfister and Fertig 2010; Fernihough 2012; Herzer et al. 2012; Moller and Sharp 2014; Rathke and Sarferaz 2014; Edvinsson 2017) have argued in favor of complementing "Malthusian effects" in the sense that the PoP in its tendency to raise population not only enhances fertility, but at the same time operates towards lower mortality. These authors regard the statistically significant positive effect of GDP per capita growth on birth rate as evidence of "preventive checks" in general, which are not to be confused with the GPC. Accordingly, if income is observed to raise births on average, it is a sign that reproduction has formerly been suppressed by preventive fertility behavior. Equally, they hold the apparent negative causal relationship between GDP per capita and death rate to universally reflect "positive checks". Their idea is that whenever living standards would fall below a subsistence level, the positive checks are supposed to increase the death rate as a general result of individuals heavily competing for the remaining resources.

This effect deserves attention and could be added to the simulation to complement the mechanism of stagnation by providing another channel of population growth. In this paper, the modeling of the effect of conventional "positive checks" has been disregarded for two reasons. Firstly, it does not provide explanatory power for the mechanism of growth, since the positive checks are thought to disappear at the same time as GDP per capita rose above subsistence level. When modeling and evaluating the growth regime, it is regarded to be sufficient to focus on the steady decline of fertility as the crucial factor contributing to the population slowdown inducing the breakout from stagnation. More importantly, the current conventional interpretation of preventive and positive checks is at odds with Malthus' definition stating that

> the preventive check is perhaps best measured by the smallness of the proportion of yearly births to the whole population,[23]

i.e. by the level of the birth rate and that

> the positive checks to population [...] include every cause [...] which in any degree contributes to shorten the natural duration of life,[24]

which are best measured by the level of the death rate. Consequently, the preventive checks ought not to be measured by the causal relationship running from GDP per capita to fertility, which is reserved for the PoP. Instead, it might be very generally concluded that a low birth rate is a sign of the operation of preventive checks, whereas a high death rate reveals the operation of positive checks. Naturally, this implies an important Malthusian insight that has already been hinted at — that the regime of stagnation is characterized by high mortality and the regime of growth by low fertility.

[23] Malthus (1826), book II, ch. XI.
[24] Malthus (1826), book I, ch. II.

4 Conclusion

The purpose of this work was to provide and validate a theory that solves the economic problem, or in other words, to disentangle the effects responsible for a historical regime of economic stagnation and for a regime of economic growth. Hitherto, the field of unified growth theory has attempted to offer a theoretical analysis of the relationship between the demographic transition and the economic transition to growth that is observed in the form of stylized facts. In the present paper, having retraced unified growth theory to its classical predecessor, four classical elementary principles were interpreted to account for the relevant interactions between demographic and economic variables. While the principle of diminishing returns and the principle of labor division are commonly acknowledged in economic theory in one form or another, the existence of the principle of population remains debated. Furthermore, the great preventive check has even been ignored in recent evaluations of the Malthusian model. However, when accounting for the last two principles, classical theory is found to match the stylized facts. To trigger the transition to economic growth, it proposes to reduce mortality or, what is nearly the same, to increase life expectancy. Theoretically, this effect is justified by the fact that the demographic structure resulting from such a change is much less prone to overpopulation, as a major part of the population becomes infertile.

Eventually, as it is not sufficient to construct a model that fits the stylized facts, the operation of the classical principles had to be evaluated collectively to avoid the reasonable impression of "reverse engineering". To this end, a simple VAR estimation provided a way to establish evidence of the suggested classical causalities by employing cumulative impulse response functions derived from three historical samples, based on approximately 4,500 observations on annual national data of birth rate, death rate and GDP per capita growth. In those cases, in which causalities were a priori supposed to be measurable, in particular for the great preventive check and the principle of population, the impulse responses yielded strong support. Additional robustness tests conducted with regard to country–specific effects and time–varying coefficients were generally in line with the author's interpretation of the classical principles. Potential future results from using a VEC or SVAR interpretation instead of a traditional VAR or from employing time–varying coefficients are not expected to yield very different results. Also, it has been suggested that recent publications might require reconsideration regarding the use of "positive checks" and "preventive checks", as they seem to be at odds with Malthus' original terminology.

With the establishment of the principle of population and the great preventive check, classical theory yields an explanation that can solve the "demographic economic paradox", which states that economies with higher GDP per capita tend to exhibit lower birth rates. For policy implications, it is important to realize that there exists no such negative causal link running from living standards to fertility. On the contrary, development support in the form of wealth might even aggravate the population problem.

As a consequence of the great preventive check, the most practicable and probably most human way to limit population pressure consists in a reduction of the death rate that is largely kept high by epidemics such as currently in equatorial areas. This reduction of mortality will certainly raise the population pressure on the upcoming generation for some time and correspondingly increase poverty. However, so far no instance has been observed in which decreasing fertility and increasing GDP per capita was not preceded by such a transitional period.

Notwithstanding the empirical validation of these effects, an important shortcoming of this work lies in the omission of the effects to be observed from the principle of labor division. Until they can be measured, the classical unified growth model cannot be said to have been fully confirmed by the data.

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Appendix I

Figure 4.1: England & Wales: Stagnation and growth in GDP per capita 1300–2010. Sources: Clark (2009) for 1302-1869, Mitchell (2013) for 1869-2010.

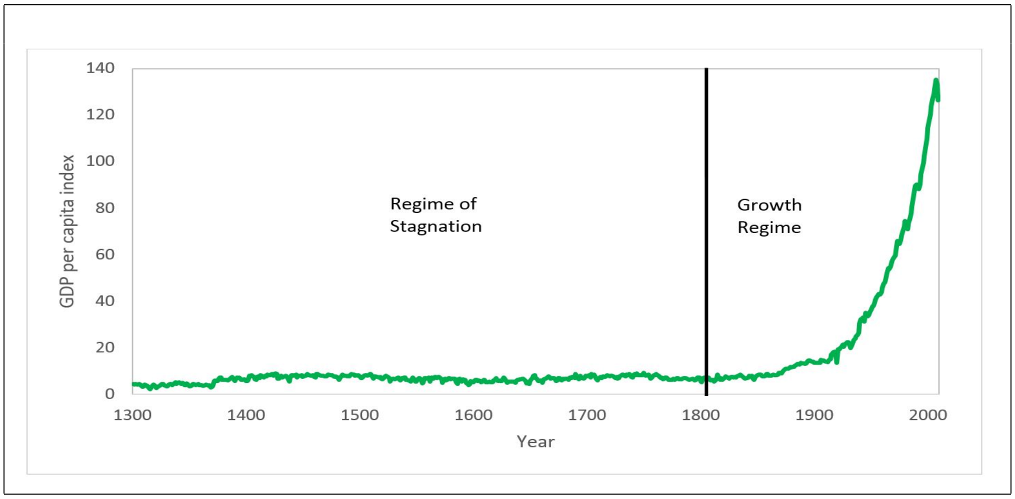


Figure 4.2: England & Wales: Stagnation and growth in London real wages 1300–2010. Sources: Allen (2001) for 1300-1869, Mitchell (2013) for 1869-1980.

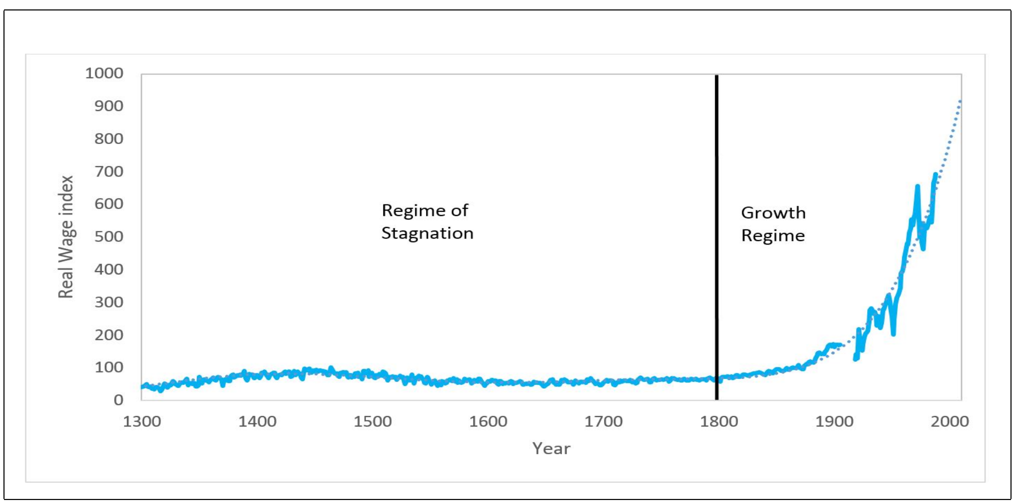


Figure 4.3: 19 selected countries: Birth rates (blue), death rates (red) and GDP per capita (green); GDP per capita is indexed to the year 2010 = 0.05, x- and y-axis intersect at value zero; arg–hun

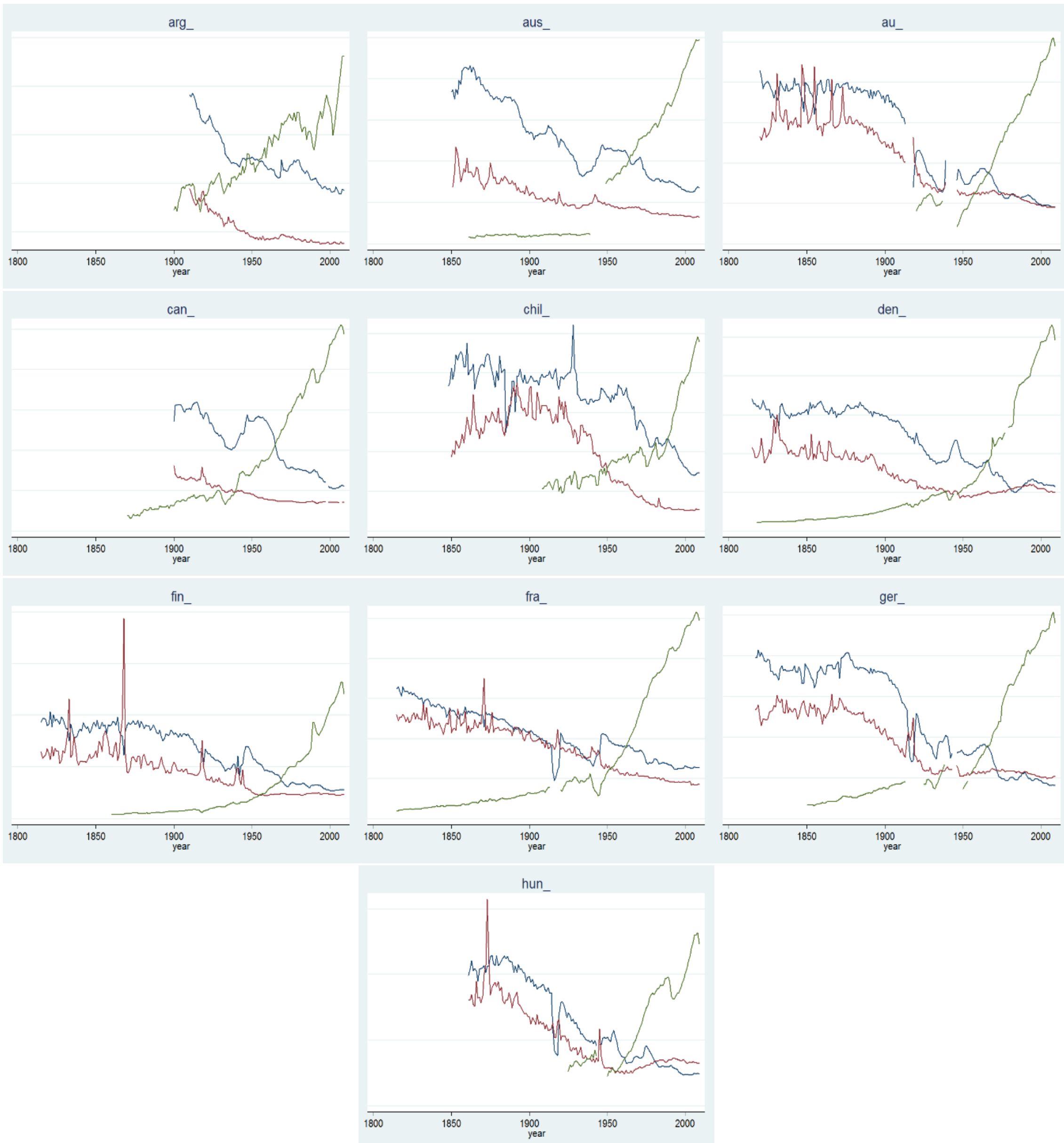


Figure 4.4: 19 selected countries: Birth rates (blue), death rates (red) and GDP per capita (green); GDP per capita is indexed to the year 2010 = 0.05, x- and y-axis intersect at value zero; ita–swi

Table 4.1: England & Wales: Lag selection criteria

Selection-order criteria						Number of obs = 459 Sample: 1552-2010		
lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-4664.97				136775	20.34	20,350	20,367
1	-3622.06	2085.8	9	0.000	1511.8	15,835	15,877	15,943
2	-3564.38	115.36	9	0.000	1222.85	15,623	15,697	15,812
3	-3533.78	61,203	9	0.000	1113.02	15,528	15,635	15.798*
4	-3506.32	54,917	9	0.000	1027.03	15,448	15.586*	15,799
5	-3496.7	19,242	9	0.023	1024.3	15,445	15,615	15,877
6	-3486.65	20,103	9	0.017	1019.67	15,441	15,643	15,954
7	-3474.39	24,521	9	0.004	1005.37	15,427	15,660	16,020
8	-3460.36	28,052	9	0.001	983,691	15,405	15,670	16,079
9	-3442.45	35,812	9	0.000	946,374	15,366	15,663	16,122
10	-3429.8	25.315*	9	0.003	931.563*	15.345*	15,679	16,187

Table 4.2: List of countries studied

CC	Country		CC	Country		CC	Country
arg	Argentina		e&w	England & Wales		net	Netherlands
au	Austria		fin	Finland		nor	Norway
aus	Australia		fra	France		nz	New Zealand
can	Canada		ger	Germany		rom	Romania
chil	Chile		hun	Hungary		spa	Spain
col	Columbia		ita	Italy		swe	Sweden
den	Denmark		jap	Japan		swi	Switzerland

Appendix II

Figure 4.5: 19 countries: Cumulative orthogonalized impulse response functions of birth rate on a one standard deviation shock in death rate, measuring the great preventive check; arg–ger

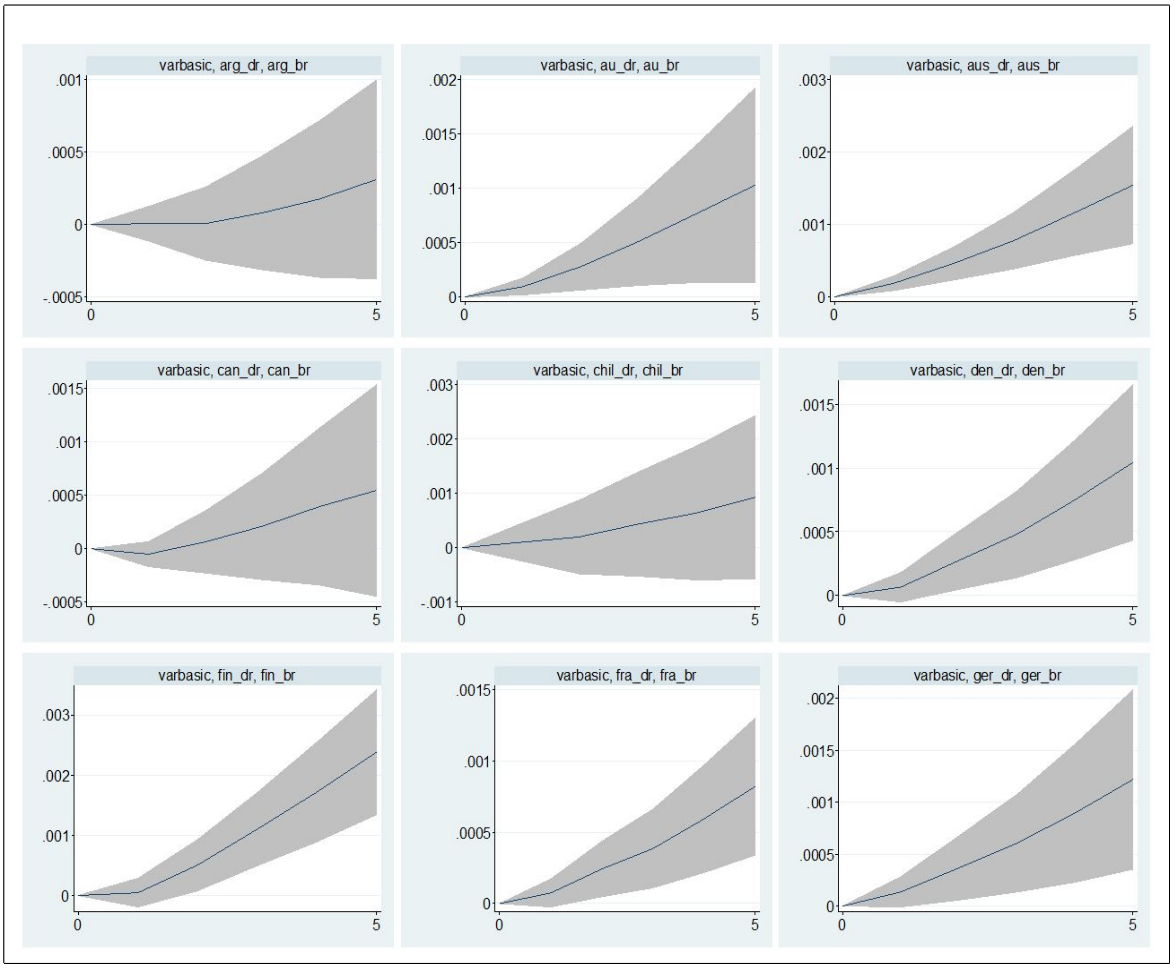


Figure 4.6: 19 countries: Cumulative orthogonalized impulse response functions of birth rate on a one standard deviation shock in death rate, measuring the great preventive check; hun–swi

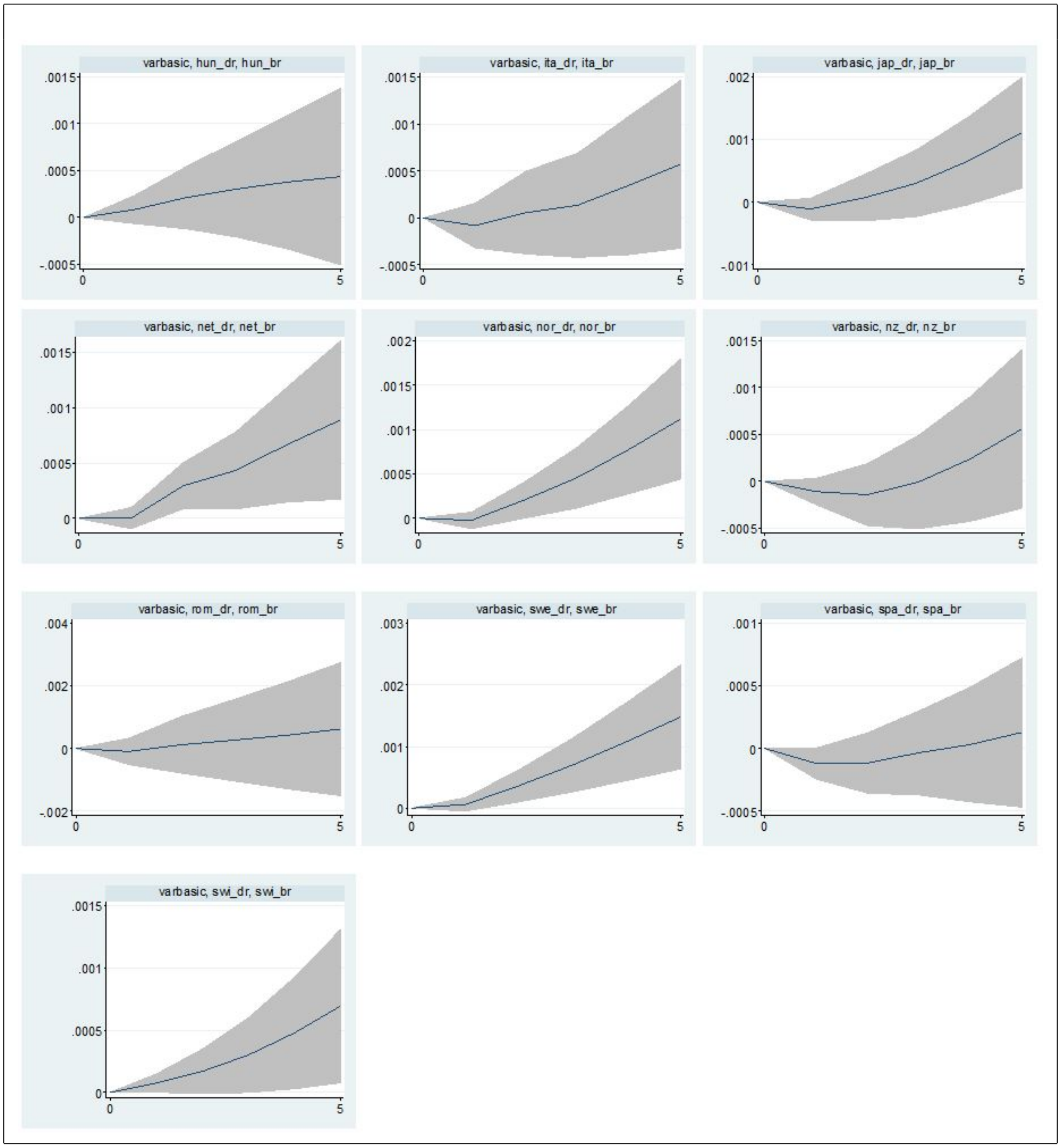


Figure 4.7: 19 countries: Cumulative orthogonalized impulse response functions of birth rate on a one standard deviation shock in GDP per capita growth, measuring the principle of population; arg-ger

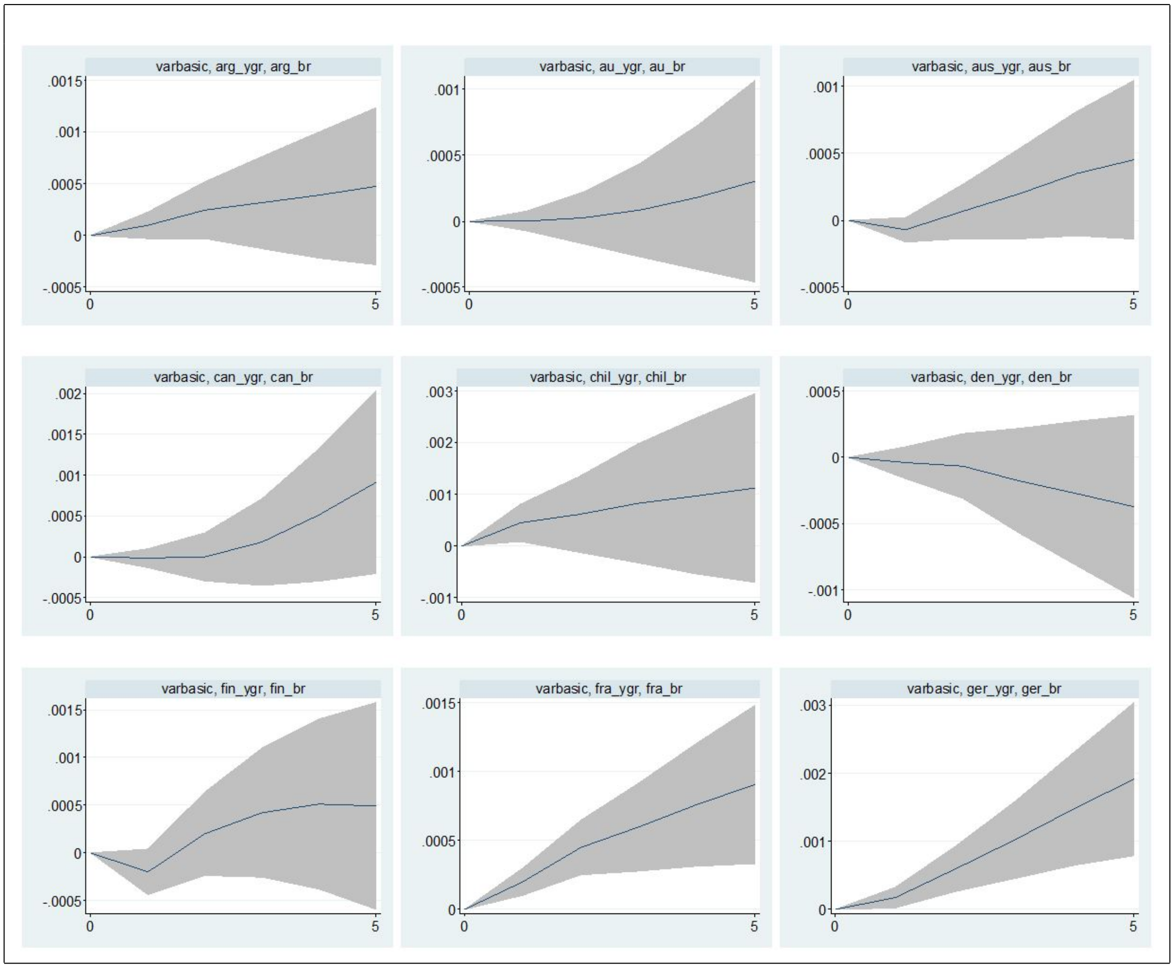
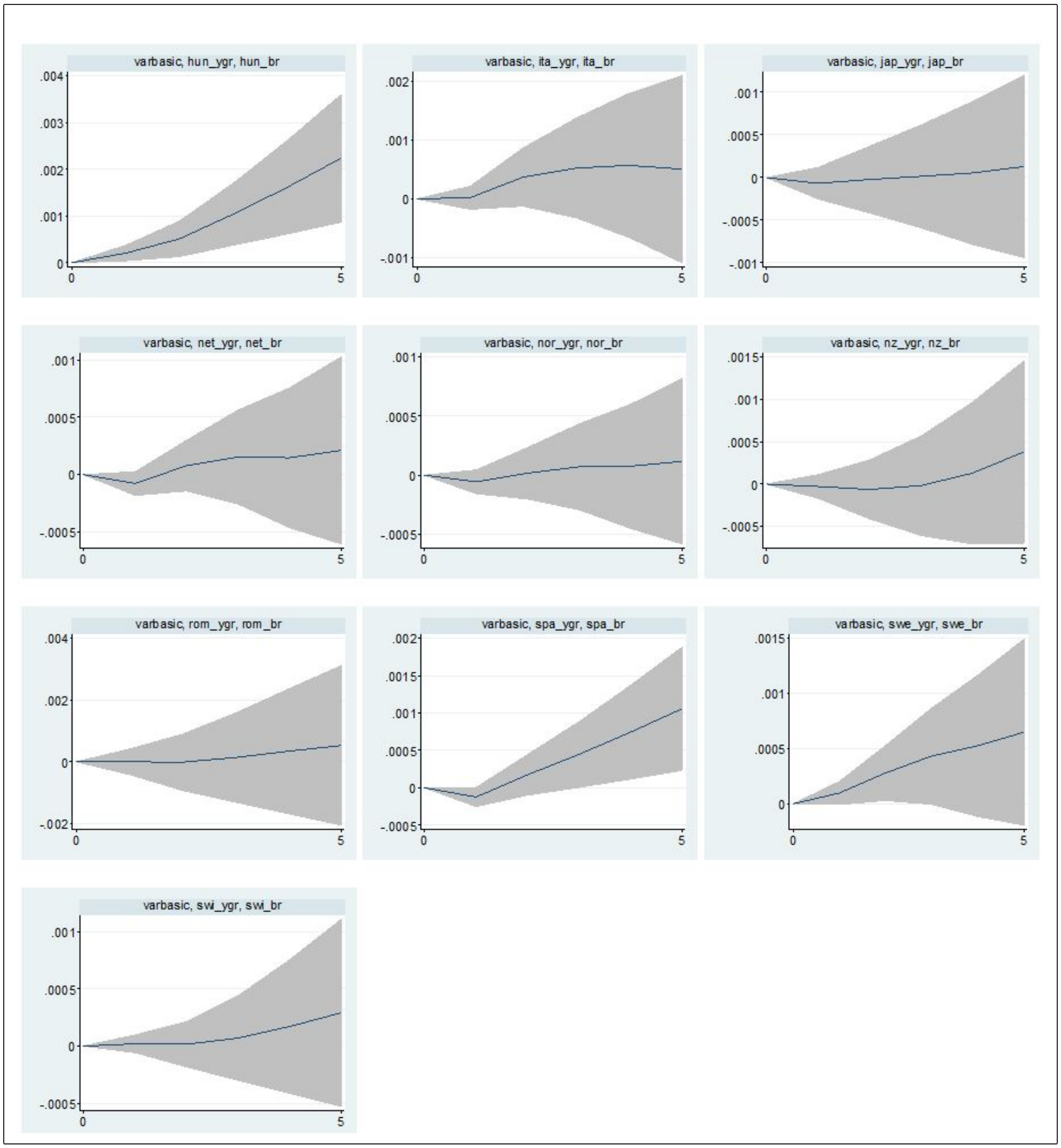


Figure 4.8: 19 countries: Cumulative orthogonalized impulse response functions of birth rate on a one standard deviation shock in GDP per capita growth, measuring the principle of population; hun-swi



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ISSN: 1438-2733